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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

THE ESTIMATION OF LANCHESTER ATTRITION RATE COEFFICIENTS FOR AN AGGREGATED COMBAT MODEL

by

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June 1976

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Estimation of Lanchester Attrition-Rate Coefficients for an Aggregated Combat Model.		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis ~~June 1976~~
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) ~~Lee,~~ Yin-chu / Lee ~~Pi,~~ Yu-kung / Pi		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 11 Jun 76
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE June 1976
		13. NUMBER OF PAGES 58 12 56p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Lanchester-type combat models
Markov-dependent fire
Estimation of Lanchester attrition-rate coefficients
Theater-level combat models

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This thesis considers the problem of estimating Lanchester attrition rate coefficients for an aggregated Lanchester type theater level combat model, BALFRAM, which has been used for various high level defense planning purposes. Several alternative coefficient estimation methodologies are examined, with their strengths, weaknesses, and problems of implementation in BALFRAM being discussed. Data requirements for coefficient estimation and approaches to aggregation are also discussed.

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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
June 1976

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ABSTRACT

This thesis considers the problem of estimating Lanchester attrition-rate coefficients for an aggregated Lanchester-type theater-level combat model, BALFRAM, which has been used for various high level defense planning purposes. Several alternative coefficient-estimation methodologies are examined, with their strengths, weaknesses, and problems of implementation in BALFRAM being discussed. Data requirements for coefficient estimation and approaches to aggregation are also discussed.

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I. INTRODUCTION

A. WHY MODEL COMBAT

A war game has been defined (8) as "a simulation, by whatever means, of a military operation involving two or more opposing forces, conducted using rules, data and procedures designed to depict an actual or assumed real life situation." It is a systematic method of studying military problems and can provide a means of gaining experience, identifying errors or shortcomings, and improving skills without paying the penalties of the real world.

One of the significant values of war gaming is that it can provide an impelling stimulus to innovation, motivation and creativeness. It establishes an environment that challenges and motivates a responsible participant. Thus different kinds of war games have been used extensively to train officers in military forces throughout the world.

Another advantage of war gaming is that a war game can be played on any hypothetical terrain which may not be areas controlled by the nation sponsoring the game, as must be the case in field tests and maneuvers that employ real military forces. Any nation may employ war gaming to assess or test military requirements and the contingencies with which military forces must be able to cope to ensure the security and survival of the nation.

B. OVERVIEW OF THESIS

This paper is composed of seven chapters. In chapter 1, war gaming is discussed and its values are described. Chapter 2 gives three different methods for modelling combat. These are war games, simulation and analytical models. Chapter 3 introduces a computerized analytical model BAIFRAM and discusses its analysis use in the Republic of China. Chapter 4 introduces Markov-dependent fire and means of calculating the Lanchester attrition – rate coefficient. Chapter 5 discusses the historical, logistic, weapon and qualitative data which are required for input to simulation or war gaming models. Chapter 6 examines several methodologies such as notional unit, firepower scores and the use of satellite models for the aggregation of forces, and the problem area to apply Bonder's approach in BAIFRAM. Chapter 7 gives some final remarks.

II. METHODOLOGIES FOR THE ANALYSIS OF COMBAT

A. WAR GAMES

A war game, as defined by Bonder (2), is a step removed from the reality of a field experiment or a field exercise wherein only teams of players representing the commanding officers and their staffs are included.

About 3000 B.C., the Chinese people invented a game called "Wei Chi" and it is still played today on a stylized map board with black and white colored stones and won by the player who succeeded in outflanking his opponent. Perhaps that was the origination of war games (22). But it was in the seventeenth century chess like games reflecting the military development came to a new age. In 1644, Christopher Weikhmann of Ulm developed a war chess called the "King's Game." It is said to have been highly regarded as an aid in military training. Since then various kinds of military chess were invented in many different countries. In 1824, a war game was played before General Von Muffling, the Chief of General Staff of Prussia. He had received the players rather coldly at first, but as the operations expanded on the map and move by move the combatants worked out their plans, the old general's face lit up and at last he broke out with enthusiasm: "It's not a game at all, it's training for war. I shall recommend it enthusiastically to the whole army (22)."

This was the beginning of the war game as a serious

military pursuit which was to spread to almost every country with military pretensions. But what is the usefulness of war games? It is not any information acquired from them, but the test they present to combatants of tactical and strategical knowledge. "The only difference from actual war is the absence of danger, of fatigue, of responsibility and of the friction involved in maintaining discipline and these factors are all important in war." said Wilkinson (1853 to 1937), who was a British military reformer.

When playing so called free war games, assessments regarding the effects of combat and other decisions were made subjectively by a control team of experienced military officers. So a high variance of the results was expected if different decision makers were used. And because it takes a long time to develop and to play a single game, it is not a feasible mechanism for analyzing a broad spectrum of system alternatives in a responsive manner to meet planning cycle requirements.

B. SIMULATION

War games can be simulated also by using computers. To develop this kind of model, the military process is studied and microscopically decomposed into basic events and activities, which are to be ordered in a logical sequence and programmed . Befpre being able to compute, the computer must be fed with data and parameters such as firing rate, kill probabilities, etc. When the start key is pressed, those events and activities of the different combat process are essentially followed in a specified sequence making decisions based upon predetermined rules. The final outcome will be printed out at the end of simulation.

Since combat processes contain a large number of probabilistic events and activities, simulation models require probability distributions for many of the input variables and generate the probability distributions for the output variables or results. If statistical sampling techniques involving the generation of pseudo random numbers are employed, the simulation is called a Monte Carlo simulation. Monte Carlo simulation models are employed in military planning studies.

Monte Carlo simulations tend to be more abstract than war games but are still much more concrete than, for example, the Lanchester-type combat models discussed below. For instance, two Marine Corps colonels fighting a war game over Cuba are a much subtler model of a real campaign than a computer trying to simulate the same thing. This is because the human brain can still perform a much wider variety of processes than the most elaborate electronic computer. A computer can handle much more data with accuracy and speed, but it cannot make qualitative judgements or deal with intangible factors such as leadership and morale.

C. ANALYTIC MODELS

Like simulations, analytic models also have no player involvement. To develop a model of this kind, we first study the combat process and decompose it into its basic events and activities. We then describe them mathematically. Finally one integrates these events and activity descriptions into an overall assumed mathematical structure of the process. By consistent mathematical operations, solutions can be obtained which will indicate the relationship between independent and dependent variables of combat effectiveness. When such a relationship can be

developed, it obviously simplifies the conduct of sensitivity analysis and provides an increased ease in interpreting the results, since the combat dynamics are contained in readily examined equations. Sometimes analytical solutions can not be obtained by appropriate mathematical techniques, but numerical approximations are often obtainable. This provides substantial reductions in cost and time for the conduct of military analysis.

Analytic models can be either deterministic or probabilistic. In the deterministic case, the same set of input values always produce the same set of output results; while in the probabilistic case, some of the input variables have probability distributions and produce different results over the output variables. Replications are desirable in probabalistic case.

Combat is a process that does not readily lend itself to measurement. The operational effectiveness of combat systems are often times edicted by military personnel. They are ot experimentally verified. Thus they should not be used as an evaluation mechanism to provide accurate, point estimate predictions of combat effectiveness for use by decision makers. They should be used only for analysis purposes so as to have a better understaning of the system dynamics. For this purpose, a large number of parametric variations of the model variables is required. As a consequence, for such parametric studies analytic models are preferred to simulations and war games.

Among many analytical models developed in the United States, BALFRAM (Balanced Force Requirement Analysis Methodology) is the one which has been used by Republic of China military personnel as a tool to analyze their contingency plans. BALFRAM will be discussed in the next chapter.

III. BALFRAM

A. GENERAL DESCRIPTION

BALFRAM (Balanced Force Requirement Analysis Methodology) is a computer war gaming model. It consists of some 10,000 FORTRAN statements. The basic program is primarily an analytical bookkeeping device which provides a framework within which problems of force requirements and capabilities can be analyzed and the effectiveness of force levels and force mixes can be evaluated. The entire program consists of two subprograms, namely, the NODH program and the DCSP program. The NODH program, when provided with the scenario geography abstracted from a hypothesized campaign environment in terms of nodal points and lines of access between nodes, will compute the matrices of minimum distance between nodes and of next nodes on the path of minimum distance. The DCSP program, when provided with inputs of force characteristics and contingency logic (tactical decision rules) will move units over the scenario geography, enabling them to engage according to the scenario and computing the outcome of the engagements according to appropriate fight laws designated by the user. The program is user oriented. It is very flexible and may be used to examine a wide range of military applications.

In order to use BALFRAM, scenario geography must first be abstracted into nodes representing geographical locations at which battles may take place and translated into congruous BALFRAM terms. Then force characteristics and the

tactical decision rules must be formulated and described through the use of BALFRAM descriptors. In certain sense, BALFRAM is not a complete war gaming model until a set of descriptors on force characteristics and contingency logic has been prepared and assembled to compose a scenario.

B. SYSTEM APPLICATION

1. System Organization

A simplified system organization of BALFRAM is shown in figure 1 as a tree type diagram.

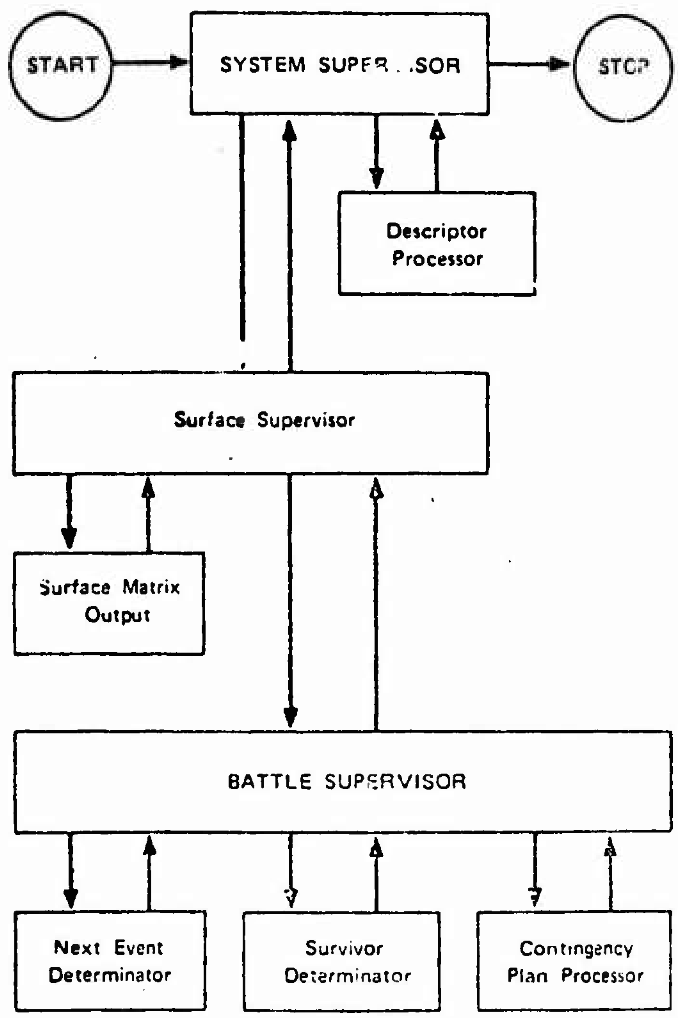

Figure 1 - SYSTEM ORGANIZATION

It shows the steps in control and execution of a scenario from the time inputs are submitted to the computation center until the time final outputs are produced along functional lines. The idea of having three supervisors each in charge of a specific step in the execution process is well conceived.

2. System Operation

In order to use BALFRAM, the user must develop a scenario and translate it into congruous BALFRAM inputs. First, scenario geography is submitted to the computation center in matrix form as inputs to NODH program. Outputs from NODH serves as geographical inputs to DCSF program. Then other inputs (force characteristics, movement logic and battle logic) are submitted to the computation center as inputs to DCSF program. Outputs from DCSF program are the final results of the scenario with respect to specific sets of parameters. BALFRAM maintains no data base except the NODH and DCSF files which are on a scenario to scenario basis. All outputs from both NODH and DCSF program are returned to the users.

C. MATHEMATICAL BACKGROUND

1. Lanchester's Classic Combat Models

The mathematical foundation of BALFRAM rests on the extention and enrichment of Lanchester's classic theory of combat between two opposing forces. Lanchester (1868 to 1946) was an English aeronautical engineer, who believed

that "one of the great questions at the root of all strategy is that of concentration; the concentration of the whole resources of a belligerent on a single purpose or object, and concurrently the concentration of the main strength of his force, whether naval or military, at one point in the field of operations [15]." To prove this point, Lanchester made a simplified mathematical analysis of the relation of opposing forces in battle, He wrote the famous "Lanchester equations" in 1914 under the assumption that:

(1) Two opposing forces each capable of inflicting casualties on the other are engaged in combat,

(2) Each unit engaged in battle are within the firing range of all other enemy units, and

(3) Both sides use aimed fire.

Then the combat between the two opposing forces was modelled by

$$dx/dt = -ay \text{ with } x(t=0) = x_0$$
$$dy/dt = -bx \text{ with } y(t=0) = y_0 \qquad (3.1)$$

where

$x(t)$=the numbers of X force at time t

$y(t)$=the numbers of Y force at time t

a=atrition rate of X force

b=attrition rate of Y force

and t= 0 denotes the time at which the battle begins.

From (3.1) Lanchester deduced his classical square law

$$b\,(x_0^2-x^2(t))=a\,(y_0^2-y^2(t)) \tag{3.2}$$

which implies that if one side committed more forces to battle at the very beginning, his casualties will be reduced significantly.

The history of force level, x(t) and y(t) were given by 18

$$\begin{aligned} x(t)&=x_0\,\mathrm{Cosh}\sqrt{ab}\,t-y_0\sqrt{a/b}\,\mathrm{Sinh}\sqrt{ab}\,t \\ y(t)&=y_0\,\mathrm{Cosh}\sqrt{ab}\,t\quad x_0\sqrt{b/a}\,\mathrm{Sinh}\sqrt{ab}\,t \end{aligned} \tag{3.3}$$

If assumption (3) is changed to be area fire, the model becomes

$$\begin{aligned} dx/dt&=-axy \text{ with } x(t=0)=x_0 \\ dy/dt&=-bxy \text{ with } y(t=0)=y_0 \end{aligned} \tag{3.4}$$

The state equation is given by

$$b\,(x_0-x)=a\,(y_0-y) \tag{3.5}$$

And the time history of, for example, the X force level is given by

$$x(t)=\begin{cases} x_0\left\{\dfrac{bx_0-ay_0}{bx_0-ay_0\exp[(ay_0-bx_0)\,t]}\right\} & \text{for } bx_0\neq ay_0 \\[2ex] \dfrac{x_0}{1+bx_0t} & \text{for } bx_0=ay_0 \end{cases} \tag{3.6}$$

2. Differential Fight Laws

Fight laws define the number of surviving components of each side as a function of time. BALFRAM contains two basic differential fight laws: the square law and the linear law. They are a modified version of the Lanchester's equations.

a. Square Law

The square law has the form

$$dx/dt=-ay-c \text{ with } x(t=0)=x_0$$

$$dy/dt=-bx-e \text{ with } y(t=0)=y_0 \qquad (3.7)$$

where $x(t)$, $y(t)$, a and b are defined as in (3.1) and c is the excgencus firepower parameter that represents the incremental capability of y to inflict attrition on x by virtue of the exogenous firepower available to y, and e is defined similarly.

The state equation is given by

$$(\sqrt{b}x_0 + e/\sqrt{b})^2 - (\sqrt{b}x+e/\sqrt{b})^2 = (\sqrt{a}y_0 + c/\sqrt{a})^2 - (\sqrt{a}y+c/\sqrt{a})^2 \qquad (3.8)$$

Analytic sulutions are always available and are given by

$$x(t)=((\sqrt{b}x_0+e/\sqrt{b})\operatorname{Cosh}\sqrt{ab}t-(\sqrt{a}y_0+c/\sqrt{a})\operatorname{Sinh}\sqrt{ab}t)/\sqrt{b}-e$$

$$(3.9)$$

$$y(t)=((\sqrt{a}y_0+c/\sqrt{a})\operatorname{Cosh}\sqrt{ab}t-(\sqrt{b}x_0+e/\sqrt{b})\operatorname{Sinh}\sqrt{ab}t)/\sqrt{a}-c$$

The differential fight laws contain the implicit assumption that this exogenous fire is aimed (rather than area) fire which is subject to discussion.

b. Linear Law

The linear law has the form

$$dx/dt=-axy-c \text{ with } x(t=0)=x_0 \qquad (3.10)$$

$$dy/dt=-bxy-e \text{ with } y(t=0)=y_0$$

where $x,y,t,c,$and e are as previously defined. The rate of attrition of each side is dependent on the number of surviving components of both sides. This is because the assumption of area fire: the more components in an area receiving uniformly distributed fire, the more casualties incurred; and the more components firing into the area, the more casualties thay will inflict.

The state equation is given by

$$b(x_0-x)=a(y_0-y) \text{ for } bc=ae \qquad (3.11)$$

In case e=c=0, analytic sulutions for equation (3.10) exist.

(a) $bx_0=ay_0$

$$x(t)=x_0/(1+btx_0) \qquad (3.12)$$

$$y(t)=y_0/(1+aty_0)$$

(b) $bx_0 \neq ay_0$

$$x(t)=\sqrt{a/b}\,r^{k}/(\exp(\sqrt{ab}\,kt)-r)$$
$$y(t)=\sqrt{b/a}\,\exp(\sqrt{ab}\,kt)/(\exp(\sqrt{ab}\,kt)-r) \tag{3.13}$$

where $r=bx_0/ay_0$ and $k=\sqrt{a/b}\,y_0-\sqrt{b/a}\,x_0$

When $c \neq 0$, $e \neq 0$ or both $c, e \neq 0$, no analytic solution exists and it is necessary to perform a numerical solution of equation (3.10).

Note: When one side uses aimed fire and the other uses area fire, one side suffers attrition at a rate proportional to only the number of firers, while the other at a rate proportional to the product of the number of firers and the number of targets. The resulting combat law is called the "mixed law." BALFRAM can also accommodate this situation.

D. BALFRAM PROCESS

Although BALFRAM is usually referred to as a computer war gaming model, it is essentially a high order compiler language. It can be regarded as model only when a complete set of inputs (descriptors) has been prepared and assembled to form a hypothesized scenario. The essential inputs consist of three categories. They are scenario geography, force characteristics, and contingency logic (tactical decision rules). Figure 2 shows the entire BALFRAM process.

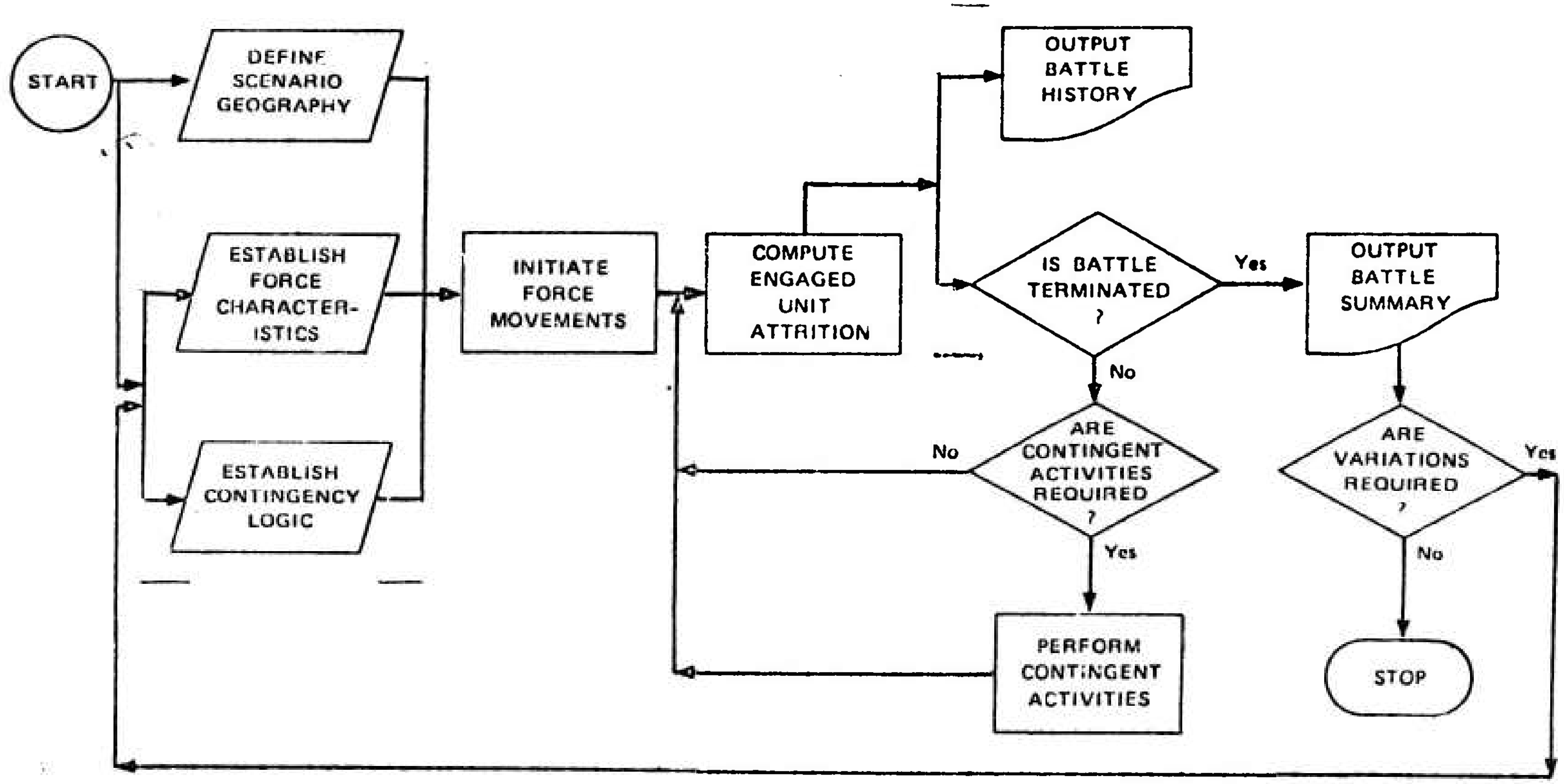

Figure 2 - BALFRAM PROCESS

1. Scenario Geography

Scenario geography is abstracted from a projected campaign environment into nodes and lines of access between them. The nodes represent defined geographic areas or specific locations. They can be located on land, on sea or even "in the air" as the scenario requires. The area represented by a node can range in size from that required for an infantry squad to that required for a battle between corps or field armies. The lines of access between nodes and their associated distances represent the lines of communication over which the forces and logistics of both sides must move. The movement of combat units during the course of battle must also follow these routes. The network of nodes can be as detailed as required by the scenario. Given a network of nodes, the NODH program will compute the shortest distance route between any two nodes in the network. Geographical irregularities such as mountains, rivers and swamps can also be input to the network for assessing the effects of force deployment and mobility on battle outcome.

2. Force Characteristics

Force characteristics refer to the number, type and nature of units each side can commit to battle, the mobility of rate at which each unit can move over the scenario geography, the combat effectiveness in terms of the ability to inflict casualties on the enemy units, and the breakpoint in term of the number of casualties each unit can sustain before being defeated. In BALFRAM, the unit is a conceptual one. It can consist of several units with each unit retaining its own characteristics or a fraction of a unit

which possess the same attrition capability, mobility and breakpoint as the unit. Units of different sizes can be input into BALFRAM as force equivalents of the standard unit which is designated by the user. As to ships and airplanes, the same principle applies. They can be input into BALFRAM either as a single ship (or airplane) or as naval (or air force) units.

3. Contingency Logic

Contingency logic refers to the sequential tactical decision rule or the way forces are to be employed during the course of campaign. A typical example of initial deployment would be what proportion of ground forces are to be deployed to front line combat positions and what proportion of forces are to be held as reserves, and the relative position or locations the front line units and the reserves are to occupy. An example of force utilization would be what proportion of tactical air force are to be used for close air support and what proportion for air to air combat. After initial deployment, it may become necessary to change deployment policy or mission allocations contingent upon some specific events which might occur. These contingent activities provide the operational priorities and movement logic which govern the way in which a unit conducts its operation as the scenario progresses. The general form of contingent logic input statement is "if some condition is true, the specified units do the following." In developing logical operations, the users are cautioned to be consistent. Inconsistencies may cause one logical step to be negated by another.

There are two types of contingency logic inputs, namely, battle logic inputs and movement logic inputs: They can be summarized as follows.

a. Battle Logic

Describe forces involved and nodes at which battles are to occur.

Specify attrition factors and criteria for battle termination.

Permit orders of battle of several units to be merged.

Specify allocation and effectiveness of supporting weapons.

Proportionally assign and redistribute forces.

Describe logistic pipelines and interdiction effects.

Vary parameters such as order of battle, firepower, mobility.

Apply principles of concentration.

b. Movement Logic

Move units from node to node contingent on arrival events.

Relocate units contingent on defeat events or at specified time.

Permit withdrawal if force ratio is unfavorable.

Cause one force to chase another.

Establish a sequential link up of forces.

Redeploy units after battle is won.

Trace movement of FEBA (forward edge of the battle area).

4. Inputs and Outputs

a. MCDH Program

Inputs to this program represent the scenario geography abstracted in matrix form with elements representing direct distance between nodes. The outputs are (1) matrix of direct distance between node pairs, (2) matrix of minimum distance between node pairs, and (3) matrix of next nodes on the path of minimum distance.

b. ICSF Program

Beside the preprocessed scenario geography inputs from MCDH program, additional inputs must be provided to construct a BALFRAM scenario. These include force levels, mobility, indices of combat effectiveness, defeat criteria, and attrition rates. The initial concept of operation must also be developed and formulated as battle and movement logic inputs. The outputs include the "battle history" and the "end of campaign summary". The former provides a chronological record of the scenario showing the location and next objectives of forces, location and status of

battles, and the size of forces involved at each battle step. The latter, surviving forces on both sides, casualties resulting from exogenous fire, and the duration of the battle. An output of sensitivity analysis results can also be obtained at the option of the user.

5. Capabilities

Treating heterogeneous forces as homogeneous, BALFRAM provides an aggregated approach to the simulation of conventional conflicts between two opposing forces composed of ground, air and naval units. It can be used to assess the capabilities of the component ground, air and naval elements in their coordinated support of a military operation or to compare the soundness of tactical decisions by examining the effects of alternative courses of action upon the outcome of a conflict. The software program can also generate game theoretic solutions to problems of force allocation and perform sensitivity analysis to quantify relationship between input and output, thus providing a framework for analyzing the tradeoffs between component forces and deriving optimal force level objectives.

BALFRAM is primarily a simulation model for a unified high command. It can be used to model combat at the theater level. There is no explicit statement on the level of force or number of units it can handle; however, the primary inputs on force characteristics and contingency logic in the form of data cards usually do not exceed 300 per scenario.

A typical example of the kind of sensitivity analysis BALFRAM can perform would be the X-force superiority, parity and Y-force superiority analysis based on the variations of Y-force levels as shown in the

following figure.

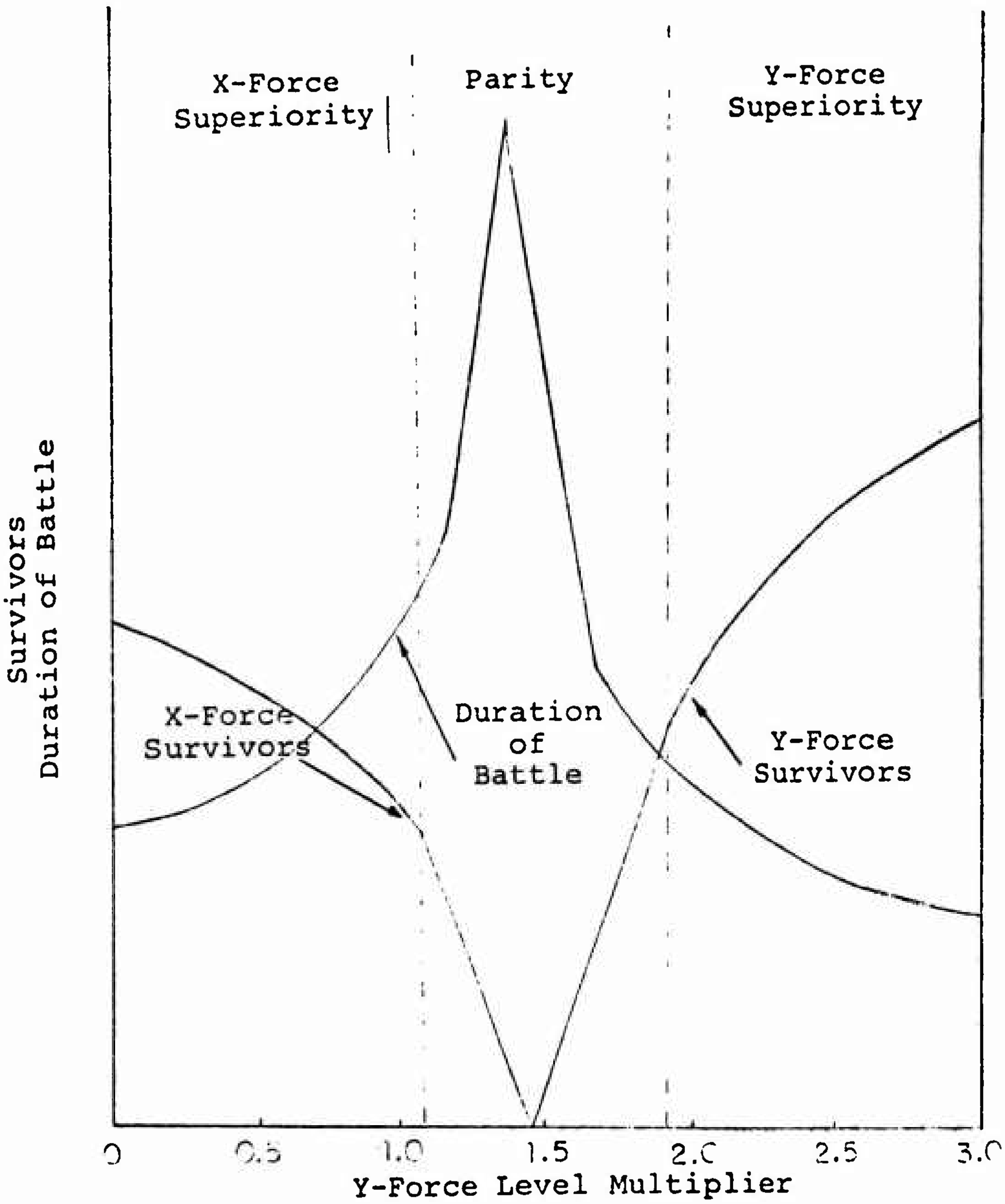

Figure 3 - SENSITIVITY ANALYSIS

E. UTILIZATION OF BALFRAM IN THE REPUBLIC OF CHINA

BALFRAM was first introduced to the Republic of China in early 1970's. Since then, BALFRAM has been used in conjunction with manual war games. Because computer war games as well as manual war games can only represent the battle field reality to some extent, playing them simultaneously using the same scenario would hopefully eliminate some of the weaknesses of both.

A major difficulty encountered in using BALFRAM was the determination of attrition rate coefficients for different types of units. BALFRAM treats units from the same Service (e.g. Army) as being homogeneous regardles of their weapon characteristics and units from different Services (e.g. Army and Air Force) as heterogeneous forces. The use of notional units in BALFRAM wherein units from the same Service are supposedly brought to equal footing by pooling and normalizing their resources by the standard unit is a step in the right direction toward the estimation of attrition rate coefficients. However, it falls short of achieving its goal because of the definition of homogeneity. An example may help to illustrate this point.

For example, if an infantry division is used as the standard unit and given a value of 1.0, i.e. one notional unit. An armored division is found to be equivalent to two infantry divisions and is given a value of 2.0, i.e. two notional units. This means that the casualty inflicting capability of an armored division is twice as that of an infantry division. However, in terms of other contributions, an armored division may be worth more than two infantry divisions. The use of armored divisions for the

exploitation of battle victories is a possible case. The defeat of France by Germany in the Second World War is an example of such an instance. On the other hand, an infantry division may be equivalent to an armored division under certain combat situations.

Due to its aggregate nature, BALFRAM's approach with respect to this problem was to leave the determination of attrition-rate coefficients to the user. It seemed to suggest that this wuold increase the flexibility and applicability of BALFRAM in that the user was free to choose the attrition coefficients and simulate many types of military operations. In view of the level of command at which BALFRAM was intended to be used, it had achieved its objectives admirably. However, for more effective use at relatively lower levels of command, BALFRAM can be improved by employing other existing methodologies to estimate attrition-rate coefficients for different types of units and under different sets of circumstances.

Another difficulty encountered was the calculation of ICE (index of combat effectiveness). Because ICE reflects the status or combat effectiveness condition of a unit (training, activation, experience, sustaining capability, etc.), the determination of ICE is primarily a matter of judgement and to a large extent a subjective one. The problem was further complicated when information regarding the enemy troops concerning these factors is incomplete and can not be relied upon. Under these circumstances, the results of the simulation could easily be tempered to please one's superior. The net result of all this would be tantamount to the negation of the purpose of the entire effort.

One way of attacking this problem would be to develop a methodology for estimating the ICE's of various types of

units with respect to quantitative and qualitative factors which have a direct bearing on the effectiveness of a unit to carry on combat. It is true that a MOE (measure of effectiveness) for such factors is hard to decide on because it is difficult to devise a acceptable mesaurement of these factors which will provide a reasonable approximation to the ICE. BALFRAM took a passive approach to this problem as in the case of attrition-rate coefficients. An active approach would be to develop a methodology and incorporate it in the program for the estimation of ICE of various types of units. This is most desirable on the part of the users.

In BALFRAM, ICE is a multiplicative factor acting upon the attrition-rate coefficient. It may not be realistic because the attrition-rate coefficient should have a diminishing marginal return with respect to increasing the ICE. A curve such as an exponential might be more suitable to describe this relationship.

IV. ESTIMATION OF ATTRITION-RATE COEFFICIENTS

In the utilization of a Lanchester-type model, the essential part is to determine numerical values for Lanchester attrition-rate coefficients from weapon system performance data. Two significant developments in this field appeared during the 1960's, namely, (1) the development of methodology for the prediction of Lanchester attrition-rate coefficients from weapon system performance data by S. Bonder [3] and C. Barfoot [1] and (2) the development of methodology for the estimation of such coefficients from Monte Carlo simulation output by G. Clark [6]. We will discuss these methodologies in this chapter.

A. MARKOV DEPENDENT FIRE

The purpose of firing a gun is to destroy a target. So the hit distribution is of interest in weapon system analysis work. If we could assume that the hit probability is the same for all rounds and each of them is independent of the others, then the Binomial distribution would be suitable for the hit distribution.

Let H be a random variable denotes the number of rounds hit the target with each round has hit probability p, then for firing n rounds, the probability that at least one round hit the target is given by

$$Pr(H \geq 1) = 1-Pr(H = 0) = 1-(1-p)^n \tag{4.1}$$

But in many circumstances this model is inadequate because fire can be adjusted and aim points for rounds are not statistically independent.

Bonder assumed that the firing process is a Markov process that is the weapon system performance depending only on the outcome of the last round fired. Define

p = Prob(hit on first round)

q = Prob(miss on round | miss on previous round)

then equation (4.1) may be revised as

$$Pr(E \geq 1) = 1-(1-p)\ q^{n-1}, \text{ for } n > 1 \qquad (4.2)$$

This paper will deal with this type of firing process and called it Markov-dependent fire.

B. BONDER'S METHOD

Recall the Lanchester-type equations for combat between two homogeneous forces

$$dx/dt = -ay \text{ with } x(t=0) = x_0$$
$$dy/dt = -bx \text{ with } y(t=0) = y_0 \qquad (4.3)$$

In this model, a and b are called the Lanchester attrtion-rate coefficient. For example, a represents the rate at which a single Y weapon system destroys X targets. It has the dimension of

(X casualties)/((number of Y units)·(unit time))

Bender has defined A, a random variable, as the rate of destruction of X target and its value is given by

$$A = 1/T \qquad (4.4)$$

where T is a random variable denotes the time for a Y firer to kill an X target. The "average" rate of target destruction would be the expected value of A, denoted by $\bar{a}$ and

$$\bar{a} = E(A) = E(1/T) \qquad (4.5)$$

However, in a paper published in 1969, Barfoot pointed out the average attrition of equation (4.5) is inadequate as well as not mathematically tractable. He suggested defining the average kill rate as the reciprocal of the expected time to destroy a target.

$$\bar{a} = 1/E(T) \qquad (4.6)$$

The reason is that in Bonder's model the probability distribution function represents the fractions of targets killed for which each rate is used, then the harmonic mean of the rates should be used. If the probability distribution function represents the fraction of the time that each rate is used, then the arithmatic mean of a set of attrition rates will be appropriate.

Equation (4.6) agrees with the intuitive definition of $\bar{a}$, the average attrition rate is the ratio of the number of targets killed in a large number of battles to the time interval over which the targets were killed. If n targets are killed and t is the time interval between which targets i-1 and i were killed, then

$$\bar{a} = \frac{n}{\sum_{i=1}^{n} t_i} = \frac{1}{\frac{1}{n}\left(\sum_{i=1}^{n} t_i\right)} = \frac{1}{\bar{t}} \tag{4.7}$$

In a later paper [4], Bender suggested a way to calculate E(T) as

$$E(T) = t_a + t_1 - t_h + (t_h + t_f)/P_K + ((t_m + t_f)/p) \cdot ((1-u)/P_K + u - p_1) \tag{4.8}$$

where

t_a = time to acquire targets

t_1 = time to fire the first round

t_h = time to fire a round, given the preceeding round was a hit

t_m = time to fire a round, given the preceeding round was a miss

t_f = projectile flight time

p_1 = first round hit probability

p_K = the conditional probability of kill, given a hit

u = conditional probability of a hit, given the preceeding round fired hit the target, and

p = conditional probability of a hit, given the preceeding round fired missed the target.

Prof. Taylor stated that if the following assumptions

can be justified,

(1) $t \cong C$

(2) $t_l = t_h = t_m = t = 1/v$, where v denotes the rate of fire,

(3) $p = r = p_l = p_h$, where p_h denotes the single shot hit probability, and

(4) $t_f \cong C$

Then equation (4.8) can be simplified as follows

$$E(T) = 1/(v \cdot p) \tag{4.9}$$

where $p = p_h p_K$

C. CLARK'S MODEL

G. Clark [6] has added another factor, target acquisition probability, into Lanchester-type combat models in his development of the COMAN (COMbat ANalysis) model.

The fundamental concept used in constructing the COMAN model is a kill rate for specified firer/target-type combinations. These kill rates are estimated from simulation data and they provide insights as to the relative effectiveness of various weapon types without resorting to numerous simulation runs.

Clark points out that for modelling purposes a target is acquired by a firer when fire can be directed towards the target's position. Acquisition can be accomplished by visually detecting the target so that its position is known

and then directing fire at the position where the target is actually located. This is the case when direct fire weapons are used. However indirect fire weapons such as artillery can be fired over hill masses that obstruct a line of sight between the firer and acquired target position. So knowledge of the exact target position can be acquired by firing at likely area for targets to be located until fire happens to be directed at an actual target position by chance.

The effects of target acquisition are introduced by using the probability that a target is unacquired as a parameter, Define

p = the probability that a specific X target is unacquired by an individual Y firer, and

q = the corresponding probability for a Y target X firer combination,

then

$$dx/dt=-ay(1-p^{x})$$
$$dy/dt=-bx(1-q^{y}) \tag{4.10}$$

where $a>0, b>0, 0\leqslant p<1$ and $0\leqslant q<1$.

It can be seen that if targets are readily acquired, equation (4.10) becomes

$$dx/dt=-ay$$
$$dy/dt=-bx \tag{4.11}$$

which is the familiar Lanchester's square law.

When individual targets become increasingly difficult to acquire that is the probability of a target being unacquired assume values close to one, the combat situation represented by equation (4.10) is equivalent to the situation envisioned by Lanchester when he formulated the linear law. This relationship is shown by expressing equation (4.10) as a function of p and q and expanded in a Taylor series about the points $p=1, q=1$

$$dx/dt=f(p)$$

$$=-ay(1-p^{x})$$

$$=-ay(f(1)+f'(1)(p-1)+f''(1)/2!)(p-1) +\dots)$$

$$=-ay(-x(p-1))=-axy(1-p)$$

similarly

$$dy/dt=-bxy(1-q) \qquad (4.12)$$

V. DATA REQUIREMENT FOR COEFFICIENT ESTIMATION

In running a simulation or war gaming model, a wide range of factors must be considered. Though simplifying parity assumptions can be made regarding common factors, quantified values must be provided to describe those factors for which differences exist between the two opposing forces. This gives rise to the problem of input data requirements, which is crucial to the estimation of attrition coefficients for different types of units under different combat situations.

The data required for input to simulation or war gaming models for the evaluation of military problems can be classified into four categories:

A. HISTORICAL DATA

Since the formulation of the linear and square laws by Lanchester in 1914, substantial amount of research work has been done in the area of mathematical modeling of combat. Efforts in this respect were primarily centered on the extention and enrichment of Lanchester's theory of combat, which has been validated to be adequate to represent the dynamic process of classic combat by:

(1) Helmbold using data on twenty seven battles which occurred in the United States between 1759 to 1945 (also several subsequent studies [10] [14]),

(2) Engel using the Iwo Jima engagement data of the Second World War,

(3) Willard using data of the land battles of the years 1618 to 1908, and

(4) Weiss using American Civil War data.

However, using historical data to estimate the parameters of Lanchester—type combat model must be very careful because:

(1) Historical data from different sources usually are not consistent,

(2) It is difficult to decide how to count reserves, reinforcement and maneuvering elements,

(3) Casualties may in some instances have been estimated by subtracting "stength after battle" from "strenth before battle". It is a value calculated from two inaccurate numbers hence large errors may by expected, and

(4) The duration of engagement is unreliable, usually it is estimated by the author.

Furthermore, along these same lines Helmbold [11] has discussed the uselessness of historical data for making future combat outcome predictions. Meanwhile, different types of simulation and war gaming models have been developed since the end of the Second World War. While war gaming techniques are being energetically extended to fields where little or no previous experience with the techniques in sophisticated form exists, essentially no parallel efforts have been made to compile and analyze data on past military engagements. This may be attributed to the

following reasons:

(1) When nations were at war with each other, they would be so engulfed in it that they could not afford to divert their effort to data collection,

(2) Though much can be and have been learned by studying the successes and failures in past wars, few nations have had the chance to fight a war in the same general situation a second time, and

(3) Even if actual combat data on past wars were available, it was doubtful whether they would be of any significant value to military analysis because of the rapid advances in science and technology. New weapons which have great impact on military doctrine and the concept of operations have been developed. The kind of war which will be fought by forces equipped with these weapons will definitely be different from past wars.

In view of the above, the value of historical data resides only in the validation of models; it has only limited usefulness in predicting the outcome of future wars. This has led to the present trend of using computer simulation and war gaming to generate data for military analysis.

B. LOGISTICS DATA

Logistics input data are particularly susceptible to quantification. Appropriate data on practically every thing that can be procured, transported, used and consumed exist in some tangible form. Items such as equipment, ammunition, food, gasoline, etc., fall into this category. Data such

as distances, means of transportation, volume and weight to be transported, time in transit, etc., are readily available and can be used as inputs. Furthermore, logistics requirement for each type of units can be established and placed within reasonable bounds. This will facilitate preparation and processing for use. However, currently it is not known how these logistic factors influence combat capability. In particular, there is no commonly accepted (or used) procedure for modifying combat capability due to logistic shortfalls.

C. WEAPON DATA

Weapon data include range, rate of fire, lethality, etc. Using these characteristics, different types of weapons or weapon systems can be converted into input data by firepower score or other appropriate methods. In most cases, information on enemy weapon characteristics is not available. Estimates are obtained based on known data and weapon characteristics of one's own forces. Thus the effectiveness of conventional weapons can be compared and evaluated. However, in the case of tactical nuclear weapons, the problem is complicated. Not only are lethal areas significant, but troop density in the area at the time, and the protective cover or exposure are also significant factors. In addition, contamination effects which deny the area to both friendly and enemy use must also be considered. Since tactical nuclear weapons have never been used in the past, their psychological effects in actual combat situation with respect to the morale and will to fight of troops involved is not known. Presumably estimates can be made, but how close these estimates can be to the real effects is not ascertainable.

D. QUALITATIVE DATA

The term qualitative data refers to such factors as discipline, motivation, courage, morale, will to fight, etc. By professional military judgement, these factors can be assigned quantified values to represent different levels or degrees using scaling method. Qualitatve standards such as outstanding, superior, good, etc., can be assigned numerical values, which can be used as multipliers to upgrade or degrade the expected performance of a unit. But factors such as the effects of shock and fatigue on personnel; the relative resourcefulness, initiative, and leadership of the opposing commanders; the results of communication or command control failure are not susceptible to quantification in that each war is unique in its own right. The impact of these factors on war outcome is immense and very difficult, if not impossible, to estimate. Probably BALFRAM is the only computer war gaming model which allows for the consideration of some of these factors. Though the inclusion of these factors in a simulation or war gaming is essential to the adequate representation of battlefield reality, unfortunately, their ultimate impact is beyond the imagination of the human mind.

The problem of data requirements for war gaming is complex and complicated. Except where definite objectives have been established and quantitative data exist in some tangible form, the problem appears to be of considerable magnitude and deserves a separate treatment.

VI. AGGREGATION OF FORCES

It was pointed out in the preceeding chapter that the validation of Lanchester's theory of combat had led to the encouragement of development of simulation and war gaming models based on the enrichment and extension of that theory. Such development was clearly in response to the growing need for such models in the field of military analysis and decision making. Now the problem which remains to be solved is the aggregation of forces when the forces involved on either or both sides are composed of more than one unit and are above division level. In the case of homogeneous forces (i.e. forces composed of identical units), the aggregation of forces is not a problem; however, in the case of nonhomogeneous forces, the problem is complicated and rather difficult. Several methodologies have been proposed and used to deal with the problem in the nonhomogeneous case. All of these methodologies represent steps in the right direction, but none provide a satisfactory solution to the problem because each methodology has its strengths and none lacks weaknesses. There is no commonly accepted methodology in existence.

A. NOTIONAL UNIT

The concept of notional unit (16) is one of the several existing methodologies being used to address the problem of aggregation of forces. The major strength of the approach is that it takes into account all the resources (personnel, equipment and weapons) of a unit. Though there are other

factors to be considered, the approach appears to provide a reasonable approximation to the problem as far as major factors contributing to the capability of a unit is concerned. However, the problem is: what values should be assigned to the different weapons or pieces of equipment in order to arrive at a reasonable approximation to the capability of a unit? Since the effectiveness of a weapon depends on the type of target against which it is used and the contribution of a piece of equipment depends on the environment in which it is employed, the concept of notional units must be applied with special care and emphasis on the type of combat in which a unit is to be engaged and the environment in which the unit will be fighting.

B. FIREPOWER SCORES

The firepower scores approach appeals to most military analysts because of its simplicity and ease of application. (see Stockfisch (17) for a further discussion of firepower scores and further references.) The relative firepower scores to be assigned to each type of weapons are supposed to be based on actual or experimental data with respect to a standard weapon. However, the problem seems elusive. How can the contribution of a certain type of weapon to a military operation be isolated and singled out from the many weapon systems involved? Given this can be done, what firepower scores should be assigned to a weapon considering the different types of targets against which a weapon may be used? For example, consider the firepower scores for a 90MM tank gun and a M14 rifle. If the M14 rifle is chosen as a standard weapon and assigned a value of 1, what value should be assigned to the 90MM tank gun? If lethality, rate of fire, mobility, protection, type of targets are considered, some basis of comparison exists and a value for the 90MM

tank gun may be derived. Military experience and expertise and familiarity with the weapons may provide a professional judgement as to the reasonableness of the estimation, but this may be as far as one can go.

C. THE USE OF SATELLITE MODELS

In view of the increased use of simulation and war gaming techniques in the analysis of military problems, it appears to the authors that the solution to the problem of aggregation of forces lies in the use of satellite models. COMANEX (COMAN EXtended) and CARMONETTE (a Monte Carlo simulation of battalion sized or lower units in ground combat) are examples of such models. COMANEX is a satellite model to be used in conjunction with CARMONETTE, a high resolution combat simulation model. Data relating to weapons characteristics, combat environment, mission, etc., for a particular mix of opposing forces are input to CARMONETTE. CARMONETTE performs a prespecified number of replications of the battle. It then outputs, for each replication, a time-sequenced casualty history identifying the time at which a casualty occurred, the casualty type and the killer type. This output is, in turn, input to the COMANEX which massages the data and outputs a set of Lanchester-type parameters which represent, essentially, the kill rates for each firer/target combination in the battle. The parameters are then used in DBM (Division Battle Model) ground combat assessments. The advantage of this approach over traditional methodology based on weapon firepower scores is that weapon and unit performance measures developed by CARMONETTE reflect variations in the combat environment, including the synergistic effects resulting from the employment of various combinations of weapons.

D. THE APPLICATION OF BONDER'S APPROACH IN BALFRAM

Though BALFRAM has the capability of handling heterogeneous forces as well as homogeneous forces, in reality it is only a homogeneous model because of the aggregated fashion in which it handles heterogeneous forces. Heterogeneous force in BALFRAM refers to units from the different component Services of the armed forces, namely, the Army, Air Force and Navy. Units are classified into types as ground, air or naval according to the Services to which they belong. Units which belong to the same Service are treated as homogeneous though they may be equipped with weapons of different characteristics such as lethality, rate of fire, range, protection, etc. BALFRAM's definition of homogeneous and heterogeneous forces is different from what is generally assumed by most operation researchers and military analysts. For example, in his mathematical models for combat between two homogeneous forces, Prof. J. Taylor defined homogeneous force to be a force composed of identical units. Gordon M. Clark defined heterogeneous force to be a force composed of weapons with different firepower, mobility, protection, and detection characteristics in the general context of land combat.

Bonder extended Lanchester's original formulation of the linear and square laws to incorporate target acquisition time, rate of fire, and conditional kill probability in the calculation of Lanchester-type attrition-rate coefficients. His definition of homogeneous and heterogeneous forces was also in the general context of land combat and agrees with those of Prof. J. Taylor and Gordon M. Clark. Bonder modeled the combat process in a detailed fashion while BALFRAM in an aggregated fashion. Bonder initially used the arithmatic mean of the time required to destroy a target as the estimate of the Lanchester attrition-rate coefficient.

However, this approach turned out to be mathematically untractable, and an explicit expression for the Lanchester attrition-rate coefficient could not be obtained. Later Barfoot suggested using the harmonic mean of the time required to destroy a target as the estimate of the average attrition- rate coefficient. Bonder modified his formulation according to Barfoot's suggestion.

Both BALFRAM and Bonder's model are based on the extention and enrichment of Lanchester's theory of combat. However, the underlying difference in the definition of homogeneous and heterogeneous forces led to different basic assumptions in the two models. As a result, the application of Bonder's approach in BALFRAM presented a greater challenge than was originally anticipated. The differences in the levels of details covered in the two models further complicated the problem. The amount of work involved is beyond the scope of this paper.

VII. FINAL REMARKS

The study of war will never become an exact science despite the striving efforts of operation researchers, scientists and military analysts. The main reason is the inability of man to predict how an individual will react in stressful and dangerous combat situation. Another reason is the vast number of variables present in a combat situation. These variables do not recur in fixed fashion, amount, degree, or weights of relative importance. Therefore, although various kinds of wars have been fought in the past and most likely will continue to be fought in the future, man's understanding of the process of war will never be adequate and complete.

In man's quest for insight into the process of war, both mathematical formulation of combat models and techniques of simulation and war gaming have been extensively used. In the area of mathematical models, substantial interest has been maintained since Lanchester first published his mathematcial theory of warfare. Among the various methods suggested for the estimation of Lanchester-type attrition coefficients, this paper considered the Markov-dependent fire and Bonder's and Clark's models in particular. In the area of simulation and war gaming, sophisticated techniques have been increasingly employed. Now war gaming and simulation have become standard practices in the analysis of military problems. Of all the existing models, this paper discussed BALFRAM in some detail and explored the feasibility of applying Bonder's methodology for the estimation of Lanchester attrition-rate coefficients in BALFRAM. It was found that BALFRAM considers theater-level

combat in a very aggregated fashion, while Bonder's approach applies to fire fights in a more detailed manner. Thus, Bonder's methodology is not directly applicable in an aggregated combat model such as BALFRAM, which models the very heterogeneous conglomeration of forces found in, for example, a division or a corps as a homogeneous force. Hence, apparently a differant approach must be used to estimate attrition-rate coefficients in such Lanchester-type force- planning models. This problem is a state-of-the-art problem in combat modelling, and further discussion is beyond the scope of this thesis.

LIST OF FIGURES

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